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An Introduction to Neural Network Methods for Differential Equations



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An Introduction to Neural Network Methods for Differential Equations

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Preface

Artificial neural networks, or neural networks, represent a technology that is rooted in many disciplines like mathematics, physics, statistics, computer science and engineering. Neural networks have various applications in the area of mathematical modelling, pattern recognition, signal processing and time-series analysis, etc. It is an emerging field for researchers and scientists in the industry and academics to work on. Also, many problems in science and engineering can be modelled with the use of differential equations such as problems in physics, chemistry, biology and mathematics. Due to the importance of differential equations, many methods have been developed in the literature for solving them, but they have their own shortcomings.

This book introduces a variety of neural network methods for solving differential equations arising in science and engineering. Emphasis is placed on the deep understanding of the neural network techniques, which have been presented in a mostly heuristic and intuitive manner. This approach will enable the reader to understand the working, efficiency and shortcomings of each neural network technique for solving differential equations.

The objective of this book is to provide the readers with a sound understanding of the foundations of neural network, comprehensive introduction to neural network methods for solving differential equations along with the recent developments in the techniques. The main purpose to write this textbook is stated in its title *An Introduction to Neural Network Methods for Differential Equations*. This book aims to get started with the neural network techniques for solving differential equations easily, quickly and pleasantly to beginners, regardless of their background—physics, chemistry, mathematics or engineering. This book is a comprehensive text on neural network methods for solving differential equations, and the subject matter is presented in an organized and systematic way. The book may serve as a background for readers who do not have in-depth knowledge of differential equations and neural networks together with building a basic skill set that can be used to be master in it. Our presentation in the book is aimed at developing the insights and techniques that are most useful for attacking new problems. To compile this book, we had to borrow

ideas from different sources and the credit goes to all the original developers of these networks; we have presented a list of references for each section.

This book has been compiled in four chapters. The Introduction provides a glimpse of the organization of the book and a general introduction. Chapter 1 consists of a brief overview of differential equations and the physical problems arising in science and engineering. Chapter 2 illustrates the history of neural networks starting from the 1940s beginning to the 1980s renewed enthusiasm. A general introduction to neural networks and learning technologies is presented in Chap. 3. This chapter also includes a description of multilayer perceptron and its learning methods. In Chap. 4, we introduce the different neural network methods for solving differential equations. The recent developments in all the techniques is also presented in this section. The conclusion is also presented at the end of Chap. 4, which concludes the topics presented in the book. An exhaustive list of references is given at the end of the book.

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